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AN ILLUSTRATIVE PROBLEM FOR THE ESIAC COMPUTER

DESIGN OF TACHOMETER STABILIZATION

In a positional control system for a heavy inertia load, the load is driven by a dc gear motor which receives its power from an Amplidyne power supply. A synchro transmitter is connected to the load and a synchro control transformer to the input control shaft. The output of the synchro control transformer is synchronously rectified in a phase sensitive demodulator consisting of a push-pull pair of 6L6 tubes using the carrier frequency for the plate supply. The output of the demodulator supplies the Amplidyne field.

The performance of the system requires that the open loop gain coefficient be adjusted to a value of 9000. The ESIAC is used to show that at this gain setting, the basic system as described above is unstable. The system will therefore be modified by the addition of tachometer feedback. The block diagram and equations for this system are shown in Figure 1.

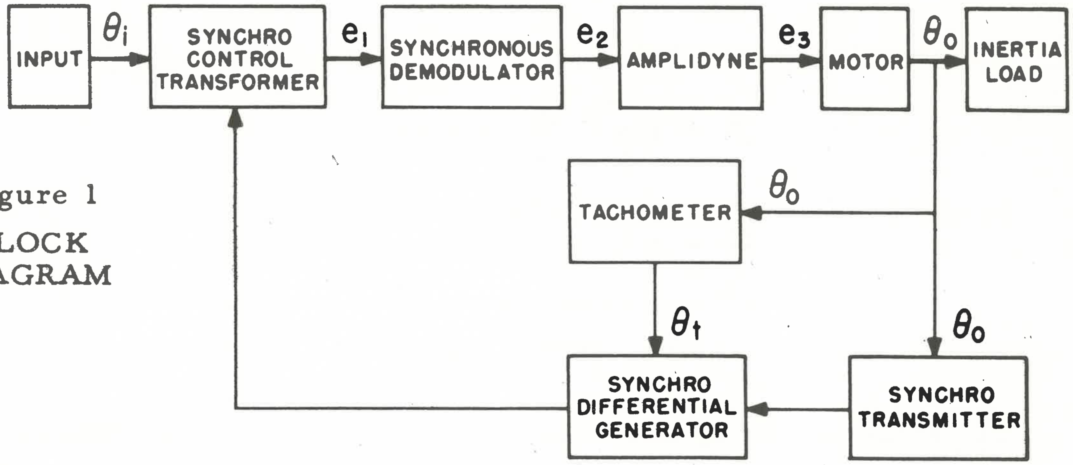

Figure 1

BLOCK DIAGRAM

Synchros:

$$e_1 = 55\,(\theta_i - \theta_t - \theta_o)$$

Demodulator:

$$e_2 = 2.27 \quad e_1$$

Amplidyne:

$$e_3 = \frac{240}{s + 12} \quad e_2$$

Motor and Load:

$$\theta_o = \frac{0.3}{s(s+6)} \quad e_3$$

Tachometer:

$$\theta_t = K_t s\, \theta_o$$

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The purpose of this analysis is to determine the optimum tachometer coefficient. The method used will be to determine first the locations of the closed loop poles of the system without tachometer feedback (when $K_t = 0$), then to determine the change in the locations of the poles as a function of K_t, selecting the value of K_t which will yield the maximum damping of the system. The locations of the poles will be determined by root locus techniques as plotted by the ESIAC.

The overall transfer function of the system is

$$\frac{\theta_o}{\theta_i} = \frac{1}{K_t s + 1 + \frac{s(s+6)(s+12)}{9000}}$$

This equation can also be written

$$\frac{\theta_o}{\theta_i} = \frac{1}{K_t s + (1 - \frac{s}{s_3})\ (1 - \frac{s}{s_4})\ (1 - \frac{s}{s_5})}$$

in which s_3, s_4 and s_5 are the roots of the equation

$$1 + \frac{s(s+6)(s+12)}{9000} = 0$$

or

$$s(s+6)(s+12) = -9000$$

This equation corresponds to the ESIAC equation in the form

$$F = K s^{n_o} \overset{i}{\Pi} (s - s_i)^{n_i}$$

in which

$F = 9000 \angle 180^\circ$

$K = +1$

$n_o = +1$

$s_1 = 6 \angle 180^\circ,\ n_1 = +1$

$s_2 = 12 \angle 180^\circ,\ n_2 = +1$

The steps in solving this equation on the ESIAC are as follows:

1. Set the controls on the right hand panel so that the equation reads

$$F = +Ks^{+1} \Pi (s - s_i)$$

2. Set $|F| = 9000$
3. Set $\angle F = 180^\circ$
4. Set $K = 1$

5. Plot points $s = 6 \angle 180^{\circ} = s_1$ and $s = 12 \angle 180^{\circ} = s_2$ on a sheet of ESIAC graph paper. (This graph paper has a logarithmic $|s|$ scale and a linear $\angle s$ scale corresponding to the ESIAC scales.) Place this graph sheet on the plotting board of the ESIAC.

6. Move the plotting stylus to point s_1 on the graph paper. Place one probe at the index point at the center of the phase sheet and another probe at the index point at the center of the magnitude sheet, using probe clamps of value 1 screwed into holes marked +.

7. Repeat step 6 for point s_2.

8. Move the plotting stylus in a small circle around point s_1 to find a point where the $\angle F$ meter indicates a null, then move the stylus away from the point along a path where the $\angle F$ null is maintained, marking any point where the $|F|$ meter also indicates a null. Each such point is a root of the equation.

9. Repeat step 9 starting from the point s_2.

10. Repeat step 9 starting from each point where an $\angle F$ null can be found along the left and right borders of the graph paper.

The plot resulting from the above procedure, with the curves along which $\angle F$ is at a null drawn in as solid lines, is shown in Figure 2.

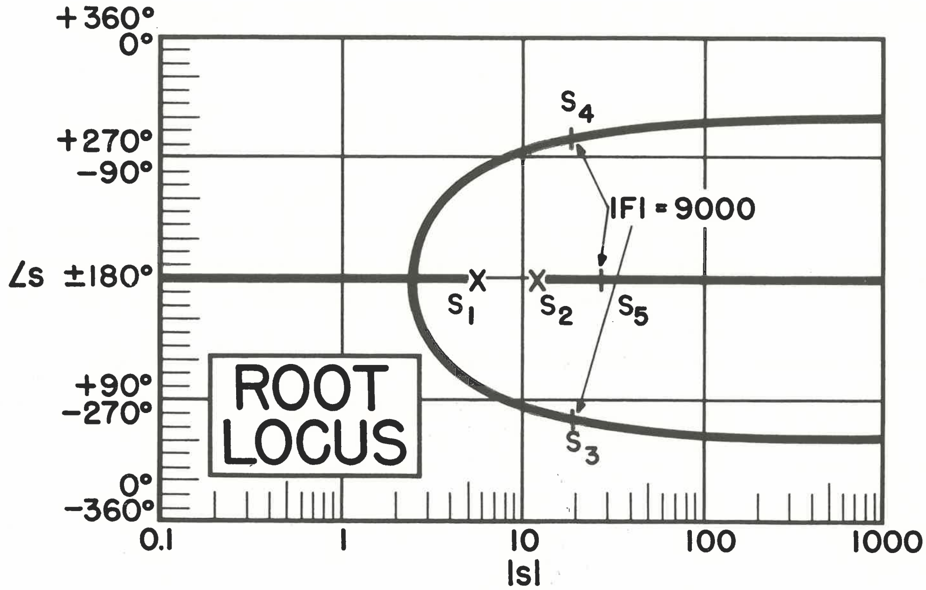

Figure 2

There are three roots to the equation, located at

$$s_3 = 18\angle 76^\circ$$
$$s_4 = 18\angle -76^\circ$$
$$s_5 = 27\angle 180^\circ$$

These roots are the poles of the system without tachometer feedback, i.e., when $K_t = 0$. It is seen that roots s_3 and s_4 represent an unstable response (an increasing oscillation), since their angles are less than 90°. Modification of the system is necessary in order to make it operable.

Using the results of the above calculation, the transfer function of the system with tachometer feedback can be written as

$$\frac{\theta_o}{\theta_i} = \frac{1}{K_t s + \left(1 - \frac{s}{18\angle 76^\circ}\right)\left(1 - \frac{s}{18\angle - 76^\circ}\right)\left(1 - \frac{s}{27\angle 180^\circ}\right)}$$

This equation can also be written

$$\frac{\theta_o}{\theta_i} = \frac{1}{\left(1 - \frac{s}{s_6}\right)\left(1 - \frac{s}{s_7}\right)\left(1 - \frac{s}{s_8}\right)}$$

in which s_6, s_7 and s_8 are the roots of the equation

$$K_t s + \left(1 - \frac{s}{18\angle 76^\circ}\right)\left(1 - \frac{s}{18\angle -76^\circ}\right)\left(1 - \frac{s}{27\angle 180^\circ}\right) = 0$$

or

$$\frac{K_t s}{\left(1 - \frac{s}{18\angle 76^\circ}\right)\left(1 - \frac{s}{18\angle -76^\circ}\right)\left(1 - \frac{s}{27\angle 180^\circ}\right)} = -1$$

This equation corresponds to the ESIAC equation in the form

$$F = Ks^{n_o} \prod^{i} \left(1 - \frac{s}{s_i}\right)^{n_i}$$

in which

$F = 1\angle 180^\circ$

K = Parameter to be determined = K_t

$n_o = +1$

$s_3 = 18\angle 76^\circ$, $n_3 = -1$

$s_4 = 18\angle -76^\circ$, $n_4 = -1$

$s_5 = 27\angle 180^\circ$, $n_5 = -1$

The parameter K_t affects only |F|, not ∠F. The method of solution will therefore be to plot the loci of values of s which yield the specified value of ∠F, then to select the points on these curves where the complex pair of roots correspond to a response with maximum damping (the points where ∠s is nearest 180°) and at these points to determine the value of K_t yielding the specified value of |F|.

The steps in solving this part of the problem on the ESIAC are as follows:

1. Set the controls on the right hand panel so that the equation reads

$$F = + Ks^{+1} \ \Pi \left(1 - \frac{s}{s_i}\right)$$

2. Set |F| = 1
3. Set ∠F = 180°
4. Lay a new sheet of ESIAC graph paper over the sheet previously plotted and mark the locations of the roots of the previous equation s_3, s_4 and s_5. Place this graph sheet on the plotting board.
5. Move the plotting stylus to point s_3. Place one probe at the index point at the center of the phase sheet and another probe at the index point at the center of the magnitude sheet, using probe clamps of value 1 screwed into holes marked -.
6. Repeat step 5 for point s_4.
7. Repeat step 5 for point s_5.
8. Move the plotting stylus in a small circle around point s_3 to find a point where the ∠F meter indicates a null, then move the stylus away from the point along a path where the ∠F null is maintained, marking a sufficient number of points along this path to establish the curve.
9. Repeat step 9 starting from point s_4.
10. Repeat step 9 starting from point s_5.
11. Repeat step 9 starting from each point where an ∠F null can be found along the left and right border of the graph paper.
12. Locate and mark the points where the curves starting from s_3 and s_4 approach nearest the line along which ∠s = 180°; this position of the roots yields maximum damping. These roots are located at

$$s_6 = 41 \angle 99^\circ = -6 + j40$$
$$s_7 = 41 \angle -99^\circ = -6 - j40$$

13. Adjust the K control so that a |F| null is indicated. Read the value of K_t(0.18). This is the value of the tachometer feedback coefficient which is the solution to the problem.

14. To locate the other root of the equation for use in analysis of the system performance, move the stylus along the path starting from point s_5 until the point is found where a |F| null is indicated. This root will be found at

$$s_8 = 5.4 \angle 180^{\circ} = -5.4$$

The plot resulting from the above procedure is shown in Figure 3.

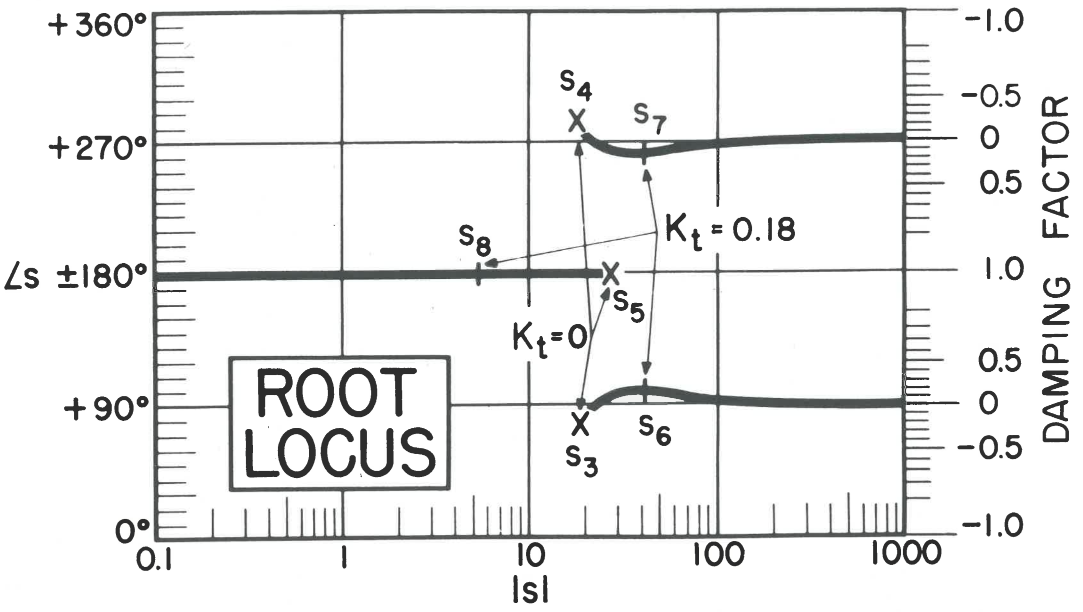

Figure 3

The overall transfer function of the system is

$$\frac{\theta_o}{\theta_i} = \frac{1}{K_t s + 1 + \frac{s(s+6)(s+12)}{9000}} = \frac{1}{\left(1 - \frac{s}{41 \angle 99^{\circ}}\right)\left(1 - \frac{s}{41 \angle -99^{\circ}}\right)\left(1 - \frac{s}{5.4 \angle 180^{\circ}}\right)}$$

As an indication of the performance with this value of K_t, the frequency response curves (gain and phase shift) are easily plotted on the ESIAC. The overall transfer function of the system when s = jω is

$$\frac{\theta_o}{\theta_i} = \frac{1}{\left(1 - \frac{j\omega}{42 \angle 99^{\circ}}\right)\left(1 - \frac{j\omega}{42 \angle -99^{\circ}}\right)\left(1 - \frac{j\omega}{5.4 \angle 180^{\circ}}\right)}$$

This equation corresponds to the ESIAC equation in the form

$$F = Ks^{n_o} \overset{i}{\Pi} \left(1 - \frac{s}{s_i}\right)^{n_i}$$

in which $|F| = \left|\frac{\theta_o}{\theta_i}\right| = \text{gain}$

$\angle F = \angle \frac{\theta_o}{\theta_i} = \text{phase shift}$

$K = +1$

$n_o = 0$

$s_6 = 41 \angle 99^\circ, \; n_6 = -1$

$s_7 = 41 \angle -99^\circ, \; n_7 = -1$

$s_8 = 5.4 \angle 180^\circ, \; n_8 = -1$

$s = j\omega = \omega \angle 90^\circ$

The steps in plotting the gain and phase shift of the system on the ESIAC are as follows:

1. Set the controls on the right hand panel so that the equation reads

   $$F = +Ks^o \; \Pi \; \left(1 - \frac{s}{s_i}\right)$$

2. Set K = 1.

3. Place on the plotting board the graph sheet on which s_6, s_7 and s_8 have been plotted

4. Move the plotting stylus to point s_6. Place one probe at the index point at the center of the phase sheet and another probe at the index point at the center of the magnitude sheet, using probe clamps of value 1 screwed into holes marked -.

5. Repeat step 4 for point s_7.

6. Repeat step 4 for point s_8.

7. Remove the graph sheet locating points s_6, s_7 and s_8.

8. Set $\angle s = 90^\circ$.

9. To plot gain, couple the plotting mechanism for |F| versus |s| . Place on the plotting board a sheet of ESIAC graph paper for gain plots. Move the plotting stylus in the |s| direction to any selected value of |s| = ω and then in the |F| direction until the |F| meter indicates a null. Mark the point. Repeat at as many values of ω as it is desired to plot for the curve. The resulting graph is shown in Figure 4.

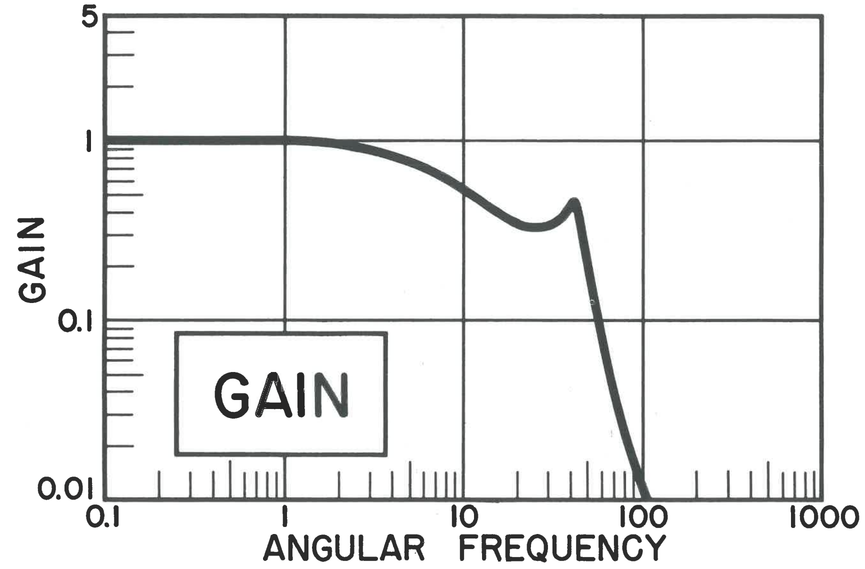

Figure 4

10. To plot phase shift, couple the plotting mechanism for ∠F versus |s| . Place on the plotting board a sheet of ESIAC graph paper for phase plots. Move the plotting stylus in the |s| direction to any selected value of |s| = ω and then in the ∠F direction until the ∠F meter indicates a null. Mark the point. Repeat at as many values of ω as it is desired to plot for the curve. The resulting graph is shown in Figure 5.

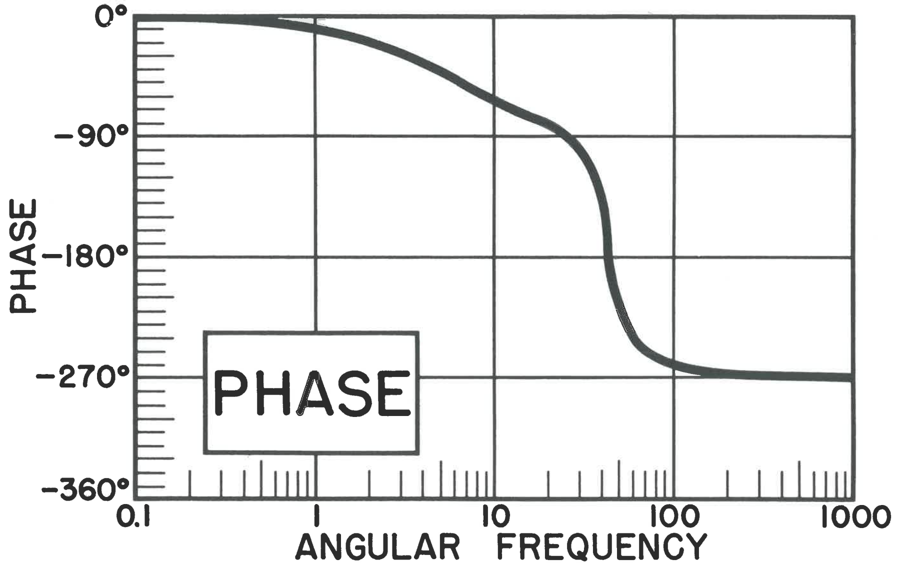

Figure 5

The time domain response for any specified input, θ_i, may be found by taking the inverse Laplace transform of θ_o. The ESIAC can be used to find the residues at the poles of θ_o and then the time domain response may be written by inspection.

For example, assuming that the input is a unit step

$$\theta_i = \frac{1}{s}$$

rewrite the transfer function

$$\frac{\theta_o}{\theta_i} = \frac{9000}{(s - s_6)(s - s_7)(s - s_8)}$$

then

$$\theta_o = \frac{9000}{(s - s_6)(s - s_7)(s - s_8)s}$$

or

$$\theta_o = \frac{A_6}{s - s_6} + \frac{A_7}{s - s_7} + \frac{A_8}{s - s_8} + \frac{A_o}{s}$$

in which A_6, A_7, A_8 and A_o are the residues at s_6, s_7, s_8 and 0

These are found on the ESIAC to be

$$A_6 = 0.07 \angle 80^\circ \quad \text{at} \quad s_6 = -6 + j40$$

$$A_7 = 0.07 \angle -80^\circ \quad \text{at} \quad s_7 = -6 - j40$$

$$A_8 = 1 \angle 180^\circ \quad \text{at} \quad s_8 = -5.4$$

and by inspection

$$A_o = 1 \quad \text{at} \quad s = 0$$

Taking the inverse Laplace transform term by term the output as a function of time is

$$\theta_t = \mathcal{L}^{-1}\theta_o = 0.14\ e^{-6t} \cos\left(80^\circ + 40t\,\frac{360^\circ}{2\pi}\right) - e^{-5.4t} + 1$$

This problem has illustrated several applications of the ESIAC which may be summarized as follows:

1. FACTORING: The fixed portion of the denominator (the part excluding the K_t term) was factored by means of the root locus technique on the ESIAC.
2. ROOT LOCUS PLOTTING: The locus of the closed loop poles of the system as a function of K_t was made on the ESIAC, and the optimum pole locations and corresponding value of K_t determined.
3. FREQUENCY RESPONSE PLOTTING: Plots of gain and phase as a function of frequency (Bode plots) for the system were made on the ESIAC.
4. TRANSIENT RESPONSE CALCULATION: The Laplace transform of the system output for a specified input was expanded as the sum of elementary terms and the coefficient of each term (the residue) was determined on the ESIAC. With this information the output as a function of time was written by inspection.